Two combinatorial optimization problems coming from transportation

Frédéric Meunier

April 3rd, 2015

CERMICS, Optimisation et Systmes

First problem

Joint aircraft-crew planning and stochastic shortest paths

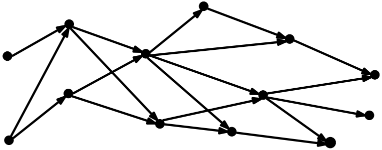
Aircraft routing and crew pairings

PhD thesis Axel Parmentier 2013-2016, Air France

Objective: compute aircraft routes and crew routes minimizing costs

Graph $D = (V, A)$:

V = flights
A = (u, v) in A if v can follow u



$$\begin{aligned} \min \quad & \sum_{p \in \mathcal{P}} c_p y_p \\ \text{s.t.} \quad & \sum_{p \ni v} y_p = 1 \qquad v \in V \\ & y_p \in \{0,1\} \qquad p \in \mathcal{P} \end{aligned}$$

$$\begin{aligned} & \sum_{r \ni v} x_r = 1 \qquad v \in V \\ & x_r \in \{0,1\} \qquad r \in \mathcal{R} \end{aligned}$$

$$\sum_{r \ni s} x_r \leq \sum_{p \ni s} y_p \qquad s \in \mathcal{S}$$

Algorithmic strategy

NP-hard problem, huge integer program : use of a **mathheuristic**.

Solve to optimality the crew pairing problem via column generation

$$\begin{aligned} \min \quad & \sum_{p \in \mathcal{P}} c_p y_p \\ \text{s.t.} \quad & \sum_{p \ni v} y_p = 1 \quad & v \in V \\ & y_p \in \{0,1\} \quad & p \in \mathcal{P} \end{aligned}$$

A posteriori check existence of a compatible aircraft routing solution via integer program

$$\begin{aligned} & \sum_{r \ni v} x_r = 1 & v \in V \\ & \sum_{r \ni s} x_r \leq \sum_{p \ni s} y_p & s \in \mathcal{S},\ p \ni s \\ & x_r \in \{0,1\} & r \in \mathcal{R} \end{aligned}$$

If not, solve again crew pairing via additional constraint

$$\begin{aligned} & z_s \geq y_p & s \in \mathcal{S},\ p \ni s \\ & \sum_{s \in \mathcal{F}} z_s \leq |\mathcal{F}| - 1 & \end{aligned}$$

Delay

Control delay: working with $p \in \mathcal{P}$ such that

$$\mathbb{P}(\text{delay along } p > \tau) < \varepsilon$$

"In $1 - \varepsilon$ of the cases, the delay is smaller than τ minutes."

STOCHASTIC RESOURCE CONSTRAINT SHORTEST PATH as a subproblem:

Input. Graph $D = (V, A)$, two vertices s and t, independent random travel times $(X_a)_{a \in A}$, costs $(c_a)_{a \in A}$.

Output. s-t path P with $\mathbb{P}(\sum_{a \in P} X_a > \tau) < \varepsilon$ and with minimum $\sum_{a \in P} c_a$.

Algorithm for Stochastic Resource Constraint Shortest Path

NP-hard problem. Idea: exhaustive and implicit enumeration of all paths.

Use stochastic lower bounds to discard uninteresting partial path.

Stochastic lower bound computed via a fixed-point equation:

$$\begin{cases} Z_t = 0 \\ Z_v = \bigwedge_{u \in N^+(v)} (X_{(v,u)} + Z_u) \quad v \in V \end{cases}$$

Experiments

Overall method currently implemented at AIR FRANCE.

Algorithm for STOCHASTIC RESOURCE CONSTRAINT SHORTEST PATH able

- to solve instances with 1600 vertices and 6500 arcs in less than 15 seconds.
- to replace $\mathbb{P}(\cdot > \tau)$ by other risk measures (such as CVaR).

`Second problem`

Shuttle scheduling

Increase the capacity of the Chunnel

PhD thesis LAURENT DAUDET 2014-2017, Chaire EUROTUNNEL

Trains in the tunnel: Eurostars, freight trains, passagers shuttles (PAX), freight shuttles (HGV)

First objective: Increase capacity in HGV's
Second objective: Increase capacity in PAX's

Constraints: Safety, "equity" schedule of Eurostars, fixed number of Eurostars and freight trains, fixed number of departures

Improvement?

EUROTUNNEL current rules: cyclic schedule, (one hour period), discretized (1 minute), 1 Eurostar every 30 minutes.

Possible way to improve...

- "Relax" these rules to improve capacity.
- Decrease waiting time of passengers.

There is room for optimization

Currently: 10 shuttles per hour, in each direction.

Elementary experiments with integer programming (CPLEX) show

- with arbitrarily small discretization: 13 shuttles per hour
- with 1 Eurostar every 25-35 minutes: 11 shuttles per hour
- with both + 2 hour cycle: 13.5 shuttles per hour.

Decrease waiting time: simplified version

⋆ One type of mobile, one directions, known cumulated demand $D(t)$ for all $t \in [0, T]$.

⋆ Objective: minimize maximum waiting time of passengers.

⋆ Constraints:

- Loading: constant rate ρ, starts at the arrival last passenger of shuttle .
- Maximal load (C) of shuttles, fixed number of departures (N).
- Safety constraints, speed limit.

Theorem

Problem polynomially solvable.

A more realistic problem

⋆ One type of mobile, two directions, known cumulated demand $D(t)$ for all $t \in [0, T]$.

⋆ Objective: minimize maximum waiting time of passengers.

⋆ Contraintes :

- Loading: constant rate ρ, starts at the arrival last passenger of shuttle .
- Maximal load (C) of shuttles, fixed number of departures (N).
- Safety constraints, speed limit.

Proposed approach

Lagrangean heuristic.

1. remove the constraints with $D(t)$ and add them to the objective function with a penalization
2. solve this problem (polynomial $\simeq$ simplified version)
3. repeat with new penalization $\longrightarrow$ convergence to a good lower bound
4. heuristically build a feasible solution from the lower bound

Experiments currently carried out.

Merci.