Gustave Eiffel University

University of Paris-Est Créteil

École des Ponts ParisTech

Master MATHEMATICS AND APPLICATIONS

Academic year 2023-2024

The “Mathematics and Applications” research master’s degree is one of the two specialties of the mathematics master’s degree at Gustave Eiffel University. It is organized in co-accreditation with the University of Paris-Est Créteil Val-de-Marne. The “Mathematics of Finance and Data” course is operated jointly with the “Mathematics and Applications” research master’s degree from the Ecole des Ponts ParisTech. This brochure describes the second year M2.

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Vallée CEDEX 2

Presentation of the second year of master's degree

The second year of the “Mathematics and Applications” master’s degree offers students dual training in analysis and probability and opportunities for specialization in various fields close to industrial applications. Students can choose one of the following four courses, the teaching unit numbers (UE) referring to the list on page 4.

Mathematics of Finance and Data Course (MFD) This course presents techniques for quantifying and hedging risks in financial markets as well as methods for data analysis and learning. We first present the mathematical tools allowing the modeling of financial securities (stochastic calculation, time series). The use of these techniques and statistical learning methods for risk valuation and management is detailed in a second step and supplemented by the transfer of experience from trading room professionals. Particular emphasis is placed on the study of probabilistic numerical methods which allow the evaluation and coverage of the corresponding financial tools, and which are also widely used in data learning. The specificities of these methods for their application to the insurance sector, interest rates, credit risk, high frequency trading and energy are detailed. This course is based on the engineering training of the Ecole des Ponts. Its numbers are limited to around twenty students (excluding students from the Ecole des Ponts).

The Mathrisk project, a joint research team at Gustave Eiffel University, Ecole des Ponts ParisTech and INRIA (National Institute for Research in Computer Science and Automation), provides scientific supervision for this course. This team is particularly developing financial risk valuation software in partnership with the professional community.

Probability and Statistics of New Data course This course is based on the probability and statistics research team of the Laboratory of Analysis and Applied Mathematics, a joint laboratory at the Gustave Eiffel and Paris-Est Créteil Universities. It presents current advances in probabilistic and statistical methods related to information processing. With the rise of massive data collection (language processing, vision, health, ecology, marketing, scoring, etc.), the scope of application of the methods presented in this course is very vast. The course places particular emphasis on recent issues in modeling, numerical simulations, statistical inference and model selection (sparsity, machine learning, etc.).

Analysis and Applications course This course is intended for students interested in all aspects of analysis and its use for modeling physical phenomena. It focuses on themes developed in the research teams at Gustave Eiffel and Paris-Est Créteil Universities. It introduces students to the most recent analysis techniques, including harmonic and Fourier analysis, multi-scale analysis and fractals, partial differential equations and the calculus of variations.

Particular emphasis can be placed, depending on individual taste and the choice of options, on the study of evolution equations (from physics and finance), on mathematical modeling, on numerical analysis and on signal processing and image analysis and synthesis.

Mathematics and Computer Science Course This course is located at the interface between mathematics and computer science, with strong requirements in both disciplines. It is based on the Labex Bézout framework common to the three mathematics and computer science research laboratories on the Paris-Est site: LAMA, LIGM and CERMICS. The timetable and control methods for this course are different from those of the three previous courses.

After a period of four weeks dedicated to consolidating the students' knowledge base in mathematics and computer science, the training continues with basic courses in discrete and continuous optimization, geometry and combinatorics and Data Science. In the second semester, two optional specialized courses can be chosen from "Deep learning", "Random matrices", "Combinatorics" and "Advanced geometry and graph theory". Classes are taught in English. For more information, see the page https://labex-bezout.fr/math-cs-track/ The four routes above are given for information only and other choices are possible. In particular, the variety of courses allows future candidates for aggregation to consolidate their mathematical culture while opening up to modeling.

Admission requirements and registration procedures

The second year of the “Mathematics and Applications” master's degree is aimed at students who have completed a first year of a master's degree in pure or applied mathematics or who have an equivalent level, as well as students from Grandes Écoles. Students are admitted on application. They must specify the course(s) they plan to follow, knowing that the number of students on the MFD course is limited to around twenty students (excluding students from the Ecole des Ponts). In the event that the information contained in the file does not allow a conclusion to be reached, candidates may be called for an interview.

Applications are made online, on the website https://candidatures.u-pem.fr/.

If you have difficulty applying by this means, contact the secretariat (Marie-Monique.Ribon@univ-eiffel.fr, tel 01 60 95 75 32).

Admitted candidates register administratively in one of the two co-authorized establishments (Gustave Eiffel and Paris-Est Créteil Universities).

An information meeting will take place on Monday September 11, 2023 at 9:30 a.m. at the University Gustave Eiffel, Copernic Building, room 2101.

Educational organization

The master's degree is organized in two semesters. Classes start on Monday September 11, 2023.

Courses in the first semester are primarily foundational courses, paving the way for the more specialized courses offered in the second semester. Improvement sessions in computer science (C++) are also planned. The second semester is devoted on the one hand to more specialized courses (from January to March) and, on the other hand, to an internship or introductory research dissertation. The list of courses given in this brochure is indicative and may be modified during the first semester, depending on the number of students and the wishes of the students.

Each course is made up of a compulsory teaching base counting for 18 ECTS. This base must be supplemented by 4 other courses at 6 ECTS each, including at least 3 in the corresponding course. The internship or end-of-study dissertation counts for 18 ECTS.

Students can, within the limit of a course of 6 ECTS, and subject to the agreement of the head of the master's degree, follow a course in other research master's degrees at the Gustave Eiffel University or even in research master's degrees. exteriors.

The introductory research course begins in April. This internship (or dissertation) can take place in a university research team or in an applied research laboratory of a public organization or company. The internship gives rise to a defense and counts for 18 ECTS (Analysis and Proba-stats course) and 15 ECTS (Finance course).

Knowledge assessment and graduation

Each course culminates in a final exam or the completion of a project. In each course, to obtain the diploma, a student must have a non-internship average of at least 10 and an internship grade of at least 10.

Opportunities

Certain courses being clearly oriented towards applications, in particular those of the MFD and probability and statistics of new data courses, students can find, at the end of the master's degree, careers in business. The application sectors concerned are finance and insurance (quantitative analysis, risk assessment, model validation, structuring, etc.), statistical data processing (web marketing, insurance, etc.), development problems. from physics. In these sectors, the needs are significant within research organizations, large industrial companies, insurance companies and banks.

Some students, in particular those who are planning a career as a researcher or teacher-researcher, may opt to prepare a thesis. The thesis can be prepared in one of the research teams associated with the master's degree (the Laboratory of Analysis and Applied Mathematics (UMR 8050 CNRS) of the Gustave Eiffel and Paris-Est Créteil Universities and the CERMICS, Teaching and Research Center in Mathematics, Computer Science and Scientific Calculation from the École des Ponts).

For graduates admitted to prepare a thesis, various funding can be considered (research grants from the Ministry of Higher Education and Research, CIFRE scholarships, scholarships from the École des Ponts, etc.). Research allocations from the Ministry of Higher Education and Research are awarded through doctoral schools. The master's degree has privileged relations with the Mathematics and ICT doctoral school of the University of Paris-Est Research and Higher Education Center.

List of EU (detailed content on the following pages)

Unit of the Mathematics of Finance and Data course (MFD)

MFD-1 Common core finance (27 ECTS)

- MFD0 Opening Week Quantitative Finance
- MFD0 Introduction to C++
- MFD1 Stochastic calculation
- MFD2 Arbitrage, volatility and portfolio management
- MFD3 Monte Carlo Methods and Stochastic Algorithms
- MFD4 Interest Rate Models

MFD-2 In-depth financial mathematics (18 ECTS)

You must validate 3 courses worth 6 ECTS, including at least 2 among:

MFD5 High Frequency Data in finance

MFD6 Credit risk

MFD7 Risk measures in finance

MFD8 Volatility Models

MFD9 Numerical methods and structured products in actuarial science

MFD10 Statistical learning and applications

MFD11 Introduction to Malliavin Calculus and numerical applications in finance

A3 Deterministic and probabilistic approximation methods for modeling applications stochastic and financial sation

Unit of the Probability and Statistics of New Data course

PS1 Common Core Probability and Statistics (12 ECTS)

You must validate at least 2 courses worth 6 ECTS among the following three units:

P1 Big data architectures

P2 High-dimensional statistics

MFD1 Stochastic calculation

PS2 Advanced courses (30 ECTS)

You must validate 5 courses at 6 ECTS among:

P3 Simulation and copulas

P4 Large random matrices and applications

A3 Deterministic and stochastic approximation methods

ACT Empirical estimation - extreme values

MFD3 Monte Carlo Methods and Stochastic Algorithms

MFD10 Statistical learning and applications

MFD11 Introduction to Malliavin Calculus and numerical applications in finance

MI Advanced Data Science

X Probabilistic and statistical modeling for epidemiology

Unit of the Analysis and Applications course

AA1 Common Core Analysis (18 ECTS)

A1 Analysis tools and partial differential equations

A2 Geometric measurement theory and multi-scale analysis tools

AA2 In-depth analysis and applications course (24 ECTS)

You must validate 4 courses worth 6 ECTS, including at least 2 among:

A3 Deterministic and probabilistic approximation methods for modeling applications stochastic and financial sation

A4 Analysis and metric theory of numbers, applications to the study of the performance of mo- communication delays

A5 Introduction to Gamma-convergence

A6 Partial differential equations and fractional Laplacian

MFD1 Stochastic Calculation

First semester

Teachers: Vlad Bally and Damien Lamberton

The aim of this course is to present the usual continuous-time stochastic processes and their main properties. These processes make it possible to model, for example, the price of financial securities.
The link with Monte Carlo methods, applications in finance and partial differential equations will also be discussed.

— Brownian motion: construction, regularity and properties of trajectories.
— Continuous time martingales, stopping time and stopping theorem.
— Quadratic variation, stochastic integral and Itô formula. — Stochastic differential equations with Lipschitz coefficients. Links with partial differential equations: Feynman-Kac formula.

Bibliography:

— N. Bouleau, Stochastic Processes and Applications, Hermann (1988).
— F. Comets, M. Meyre, Stochastic calculation and diffusion models, Dunod (2006).
— J. Hull, Options, futures and other derivatives, Prentice Hall (2006).
— I. Karatzas, S. Shreve, Brownian motion and Stochastic Calculus, Springer-Verlag (1987).
— D. Lamberton, B. Lapeyre, Introduction to Stochastic Calculation Applied to Finance, 2nd edition, Ellipses (1997).
— R. Portait, P. Poncet Market finance, 2nd edition, Dalloz (2009). Springer (1997).
— D. Revuz, M. Yor, Continuous martingales and Brownian motion, Springer-Verlag (1991).

Required prior knowledge: Measurement theory and probability calculation (see, for example, the book The Essentials of Probability Theory by J. Jacod and P. Protter, Vuibert, 2003).

MFD2 Arbitrage, Volatility and Portfolio Management

First semester

Teacher: Damien Lamberton

The aim of this course is to present the main quantitative methods for valuing derivative products and choosing optimal investments in an uncertain universe, modeled by continuous time processes. Model calibration issues and digital valuation methods will also be presented. The assumptions underlying the valuation methods and modeling choices will be highlighted and their realism will be discussed.

— Arbitrage theory: comparison of portfolios and Call Put parity.
— Study of the binomial model, risk-neutral valuation and option hedging.
— Study of the Black Scholes model: valuation using Monte Carlo methods and PDE. Construction of the hedging portfolio.
— Methods for estimating and calibrating volatility. Smile of volatility. Volatility models local and stochastic.
— Introduction to stochastic control: dynamic consistency and Hamilton Jacobi equation Bellman.
— Expected utility theory and applications to investment choices in an uncertain environment.
— Stopping problems: approximation of the maximum of a portfolio and valuation of improved options Ricans.
— Addition of frictions to the markets: special case of the addition of transaction costs or portfolio constraints.

Bibliography:

— D. Lamberton, B. Lapeyre, Introduction to Stochastic Calculation Applied to Finance, 2nd edition, Ellipses (1997).
— N. El Karoui, E. Gobet, The stochastic tools of financial markets: a guided tour of Einstein to Black-Scholes, (2011).
— S. Shreve, Stochastic Calculus for Finance Volume II: Continuous-Time Models, (2004).
— B. Bouchard, J.-F. Chassagneux, Valuation of derivative products - From fundamental theorems to hedging under risk constraints, Economica (2013).

MFD3 Monte Carlo Methods and Stochastic Algorithms

First Semester

Teachers: Benjamin Jourdain.

The objective of this course is to provide an overview of Monte Carlo methods. These numerical methods based on the simulation of random variables are among the ten algorithms that had the most influence on the development and practice of science and engineering in the 20th century. Their development continues very actively motivated by applications in data science as well as in finance, reliability or molecular simulation.

Part I: Monte Carlo methods for calculating integrals in R not

1. Empirical average of independent and identically distributed random variables: convergency and confidence intervals.
2. Variance reduction methods: control variables, importance function, analysis techniques stratification, conditioning, ...
3. Sequences with weak discrepances: theoretical elements, classic examples (Halton, Faure, Sobol, Niederreiter, ...).
4. Calculation of conditional expectations (regression, quantification, etc.) and application to the calculation of op-American tions.

Part II: Stochastic algorithms

1. Robbins-Monro algorithm: applications to optimization and variance reduction.
2. Monte Carlo methods using Markov chains: Metropolis-Hastings algorithm and annealing simulated.
3. Particle methods for filtering and simulating rare events.

Part III: Simulation of stochastic processes

1. Discretization of stochastic differential equations: classical schemes (Euler, Milshtein), convergence speeds, extrapolation techniques.
2. Model simulation with jumps.

Bibliography:

—Paul Glasserman. Monte Carlo methods in financial engineering, volume 53 of Applications of Mathematics (New York). Springer-Verlag, New York, 2004.

—Emmanuel Gobet. Monte Carlo methods and stochastic processes: from linear to non-linear, Éditions de l'École Polytechnique, 2013.

— Bernard Lapeyre, Etienne Pardoux, and Rémi Sentis. Monte Carlo methods for transport and diffusion equations, volume 29 of Mathematics & Applications (Berlin). Springer-Verlag, Berlin, 1998.

MFD4 Interest Rate Models

Second Semester

Teachers: Vlad Bally, Aurélien Alfonsi and Christophe Michel.

The aim of the course is to provide students with an introduction to the usual models used in the theory of interest rates. Three classes of models have emerged. The oldest point of view explains the behavior of interest rates by the short (instantaneous) rate. A multitude of models for short rate dynamics have been proposed, one of the main motivations being their diverse suitability for calibration. But short rate models have the disadvantage of not being able to explain the evolution of zero coupons in all generality. A new generation of models has appeared: first of all, the Heath-Jarrow-Merton (HJM) model, based on forward rates, which carries out modeling in complete generality and also has virtues from the point of view of calibration.

Then, the “market models” - that of Brace-Gatarek-Musiela (BGM), but also that of Jamishdian - which focus their interest on a certain type of financial product and establish a modeling in which the calculation of the price of this type of product is done by explicit formulas.

Course Map

Part 1. Short rate models. has. General presentation: zero coupons, short rates, instant forward rates. b. The structural equation. EDP approach and martingale approach. vs. Common models of short rates: Vasicek, Ho and Lee, Hull and White, Cox-Ingersol-Ross. d. Multi-factor models. e. Affine structure models.

Part 2. Heath-Jarrow-Merton Model (HJM). has. Martingale modeling and HJM drift condition. b. Change of currency and forward probabilities. vs. Black's formula.

d. Evaluation of the price of current products: Caps, floors, swaps and swaptions. Swap rate.

Part 3. Market models. The Brace-Gatarek-Musiela model (BGM).

Bibliography:

— Björk T. (1998), Arbitrage Theory in Continuous Time, Oxford University Press.

— Björk T. (1997), Interest Rate Theory, in Runggaldier (ed.) Financial Mathematics, Springer Lecture Notes in Mathematics 1656. Springer Verlag, Berlin.

— Brigo D. and Mercurio F., Interest rate models, theory and practice, Springer Finance, 1998.

— Lamberton D., Lapeyre B. (1997), Introduction to Stochastic Calculation Applied to Finance, 2nd edition, Ellipses.

MFD5 High frequency data in finance

Second Semester

Teachers: Aurélien Alfonsi and Sophie Laruelle.

This course initially focuses on the statistical issues of estimating volatility in the presence of Microstructure noise on financial markets. This noise is due to too frequent observation of stock prices. We will study classic statistical methods for estimating volatility parameters, and see how to adapt classic methods to the presence of this noise.

Secondly, the course will present strategies for liquidating large volumes of securities in financial markets, which requires the consideration of specific models in which a transaction has an impact on the price of securities. Indeed, when placing an order of significant size on the market, it is necessary to take into account its impact on the quotation price. In particular, the cost of its execution is no longer simply proportional to its volume. To limit its impact and cost, it is generally preferable to divide this order into several smaller orders.

To understand and quantify this, we will present "price impact" models in which we will seek to identify optimal execution strategies. We will start with the linear model of Bertsimas and Lo and Almgren and Chriss before considering more sophisticated models.

The study of these models will naturally lead us to discuss non-arbitrage conditions on short time scales, as well as the strategies used by "market makers".

MFD6 Credit risk

Second semester

Teachers: Loïc Brin and François Crénin

The lesson " Credit risk " prepares students to join a quantitative department (Grande bank: investment bank or Risk Department, financial institution). The educational approach is structured around the following three objectives:

— explain the banking environment and more particularly the issues of risk management (credit) in a bank. Make the link with the 2007 financial crisis and reforms regulatory (Basel III),

— train students in the most traditional credit instruments (bonds, securitization): in understand the risks, measure them (modeling and simulation techniques, stress tests), quantify them (pricing), manage them (credit derivatives),

— prepare students for their future career by offering them a type project on current topics, " Research and Development " working in small groups, in project mode.

Module program: http://defaultrisk.free.fr/

— Session 1: A look back at the financial crisis. Defect modeling: ratings, structural models Black-Scholes derivatives

— Session 2: Individual credit risk: the intensity view) " market" (pricing of bonds and CDS, models

— Session 3: Credit risk on a loan portfolio; correlation, dependence, model Vasicek

— Session 4: Structured products on underlying credit portfolios; CDO, CSO

— Session 5: Risk measurements, VaR

— Session 6: Stress testing, economic and regulatory capital. Towards Basel III

— Session 7: Review - project roundtables

Terms:

The course includes 7 sessions of 3 hours. The first 6 sessions are divided into a lecture course two hours followed by one hour of tutorials. The last session is devoted to round tables on student projects. The different groups, who chose their project previously from the list proposed by the teachers, take stock of the understanding of the subjects and define the objectives of the project with the teachers. The project defense takes place between March and early April at the latest.

MFD7 Risk measures in finance

First semester

Teachers: Aurélien Alfonsi and Lokman Abbas-Turki

Risk management is at the heart of the banking world's concerns, as evidenced by the recommendations of the Basel Committee on banking supervision (National Convergence of Capital Measurement and Standards). The implementation of the recommendations also results in recruitment in the banks' risk control departments. The aim of this course is to present in a theoretical part the risk measurement tools relating to the trading room and asset portfolio management. The main theoretical themes will be: monetary risk measures and the representation of convex risk measures, the theory of extreme values and the multidimensional representation of risks via copulas. In a second practical part, speakers from Société Générale will present the methods used by the different departments to assess financial risk.

The program can be consulted on the site: http://cermics.enpc.fr/~alfonsi/mrf.html.

The course is financed by the “Financial Risks Chair” of Société Générale, Ecole Polytechnique and the Ecole des Ponts. It is common with the Master in Probability and Applications of Paris 6.

Bibliography:

— Basel Committee on Banking supervision. International convergence of capital measurement and capital standards.

— Föllmer H. and A. Schied (2004) Stochastic finance. An introduction in discreet time. De Gruyter Studies in Mathematics 27, 2004.

— McNeil AJ, R. Frey and P. Embrechts Quantitative risk management. Concepts, techniques and tools. Princeton Series in Finance, 2005.

— Roncalli T. Financial risk management. Economica. 2004.

MFD8 Volatility Models

Second semester

Teacher: Julien Guyon

This course is about volatility and volatility modeling in finance. After introducing the different types of volatility, we will explain how and why volatility modeling has evolved over the years, from Black-Scholes to local volatility to stochastic volatility to local stochastic volatility to rough volatility and path-dependent volatility. We will in particular cover the following topics: the volatility smile; the links between instantaneous volatility and implied volatility; the calibration of volatility models; multi-asset volatility modeling.

Attempt agenda: - The
different types of volatility - The different
types of volatility derivatives - The volatility smile - Volatility
modeling: a brief history -
Static v. dynamic properties of volatility models
- Black-Scholes - Local volatility - Stochastic volatility - Variance
curve models
(second generation
of stochastic volatility
models)
- The smile of variance curve models - Local
stochastic volatility - The particle
method for smile calibration - Path-dependent volatility -
Rough volatility - From spot volatility
to implied volatility -
Multi-asset volatility modeling: local volatility/correlation,
stochastic volatility/correlation, path-dependent volatility/correlation, cross-dependent volatility/correlation

The course (8 readings + 1 final exam) will be taught in French or English, as per the students' preference.

This course takes place at the Ecole des Ponts.

MFD9 Numerical methods and structured products in actuarial science

Second semester

Teachers: Jacques Printems and Ludovic Goudenège

This course presents an overview of financial product structuring techniques linked to actuarial issues in the world of insurance (Variable Annuities, etc.). These specific products require their own valuation methodologies and will be presented in detail. In particular, we will study techniques based on related numerical methods for numerical resolution of partial differential equations. The course will be enhanced with practical examples illustrating the use of these techniques.

In addition, insurance companies are subject to their own solvency constraints imposed at European level. For this reason, the capital calculations necessary to deal with possible depreciation of a company's outstanding assets raise issues of generating economic scenarios, calculating quantiles and using advanced Monte Carlo methods (nested Monte Carlo) which will be presented in this course.

MFD10 Statistical learning and applications

First semester

Teachers: Romuald Elie, Jérémie Jakubowicz and Jean-Yves Audibert

The aim of the course is to present the main theoretical methods of statistical learning as well as a broad spectrum of their applications, in particular for the management of large databases. The course will be punctuated by presentations from professionals from the world of data, who will present operational applications of these methods in the fields of actuarial science, finance and web marketing.

— Theoretical foundations of statistical learning: notion of risk and empirical risk — Logistic regression and classification — Vapnik dimension, choice of
regression base — Method of k nearest neighbors,
convexification of item risk Neural networks — Update day of weighting of estimators and application in portfolio management.
— Examples of applications in Actuarial science, Finance and web marketing.

MFD11 Introduction to Malliavin Calculus and numerical applications in finance

Second semester

Teacher: Vlad Bally

The aim of the course is to give an elementary introduction to Malliavin Calculus and its applications, with a particular interest in numerical applications in finance. Students are expected to have completed a basic stochastic calculus course. The main points of the course will be as follows.

- General presentation. General integration by parts formula and applications. Differential operators and the duality formula: finite-dimensional case and passage to the limit. Clark-Ocone representation formula and coverage calculation. Applications to broadcasts.
— Sensitivity calculations. The Greeks.
— American options. Calculation of conditional expectation. Dynamic programming and the Monte Carlo method. Localization (variance reduction).
— Sobolev spaces on the Wiener space.
— Decomposition into chaos.

Bibliography:

— D Nualart (1995), The Malliavin calculus and related topics. Springer Verlag.
— N. Ikeda and S. Watanabe (1989), Stochastic Differential Equations and Diffusion Processes, North Holland.
— S. Watanabe (1984), Lectures on Stochastic Differential Equations and Malliavin calculus. Tata Institute of Fundamental Research, Springer Verlag.

P1 Big Data Architecture

First semester

Teacher: Roland Trosic

The aim of the course is to present Big Data architectures and to present the contributions of these technologies for businesses.

1. From "classic" information systems to Big Data 2. The benefits of Big Data for businesses 3. Big Data architectures in business 4. Documentary and distributed databases 5. Analysis and visualization tools 6. Machine learning 7. Big Data project development methodology

The following technologies will be seen in the course: Hadoop, Json, MongoDB, Elastic Search, Microsoft Azure, ML Azure, Python.

Bibliography: •

Rudi Bruchez (2015) - NOSQL databases and Big Data , Editions Eyrolles
• Kristina Chodorow (2013) - MongoDB - The Definitive Guide 2e - O'reilly •
Lemberger, Batty, Morel, Raffaelli (2015) - Big Data and Machine Learning, Paperback •
Cointot and Eychenne (2014) - The Big Data Revolution - Dunod •
Owens, Lentz (2014) - Hadoop in practice - Paperback

Location: •

http://openclassrooms.com : online course site • http://
www.mongodb.org : institutional site of the company MongoDB • https://
www.elastic.co : institutional site of the company Elastic • https://
aws.amazon.com : Amazon cloud computing service • https://
azure.microsoft.com/fr-fr : Microsoft cloud computing service • https://cloud.google.com/ :
service Google cloud computing

P2 High-dimensional statistics

First semester

Teacher: Mohamed Hebiri

This course is made up of three parts: classic results in the statistics of non-models parametric; aggregation methods; parsimony and methods for big data.

It requires little specific knowledge of statistics but a good command of probabilities. lities and linear algebra at undergraduate level.

The points covered in this course will be as follows: I. Asymptotic speeds for the estimation of functions, minimax theory.

- Density estimation by kernel, window selection - Non-parametric regression model, estimator by projection on an orthonormal basis (for example Fourier series), selection of models II. Aggregation of estimators III. Parsimony and regression in high dimensions: - scourge of dimension,
- parsimony in the Gaussian sequence model, soft and strong thresholding - BIC and Lasso estimators

Bibliography

• Comte, F. (2017). Non-parametric estimation. Spartacus

• Tsybakov, AB (2004). Introduction to non-parametric estimation. Springer-Verlag, Berlin.

• Hastie, T., Tibshirani, R., Wainwright, M. (2015) Statistical Learning with Sparsity. Chapman and Hall

• Wainwright, MJ (2019) High-Dimensional Statistics. Cambridge University Press

P3 Simulation and copulas

First Semester Teacher:

Thierry Jeantheau.

This course is also a unit of the professional Actuarial master's degree. It is aimed at students who have already received a basic course in probability, who have already studied Markov chains, and knowledge of R software is desirable. It presents the different methods for computer simulation of random variables. The case of random vectors is also treated, and the notion of copula is introduced to model and simulate specific dependence structures. We discuss the use of data simulated by Monte Carlo methods, particularly for the calculation of integrals. We present the use of Markov chains to simulate complicated laws (MCMC method), and in particular the Metropolis algorithm. Finally, we apply this method to solve optimization problems, by presenting the simulated annealing algorithm.

The emphasis will be placed on putting these methods into practice, which will be programmed by the students, using the statistical software R.

1. Methods for simulating random variables and vectors.
2. Introduction to Copula modeling and simulation.
3. Monte Carlo methods, application to integral calculations.
4. Markov chain simulation (MCMC method), Metropolis algorithm.
5. Application to the optimization problem, simulated annealing algorithm.

P4 Large random matrices and applications

Second semester

Teachers: Olivier Guédon, Jamal Najim and Philippe Loubaton

The theory of large random matrices aims to describe the spectrum (set of eigenvalues) and the eigenvectors of matrices whose entries are random and whose dimensions jointly tend towards infinity. The first works date back to Wigner (48) in the context of symmetric matrices, then to Marcenko-Pastur (67) in the case of empirical covariance matrices. The initial motivations for Wigner and Pastur's work came from theoretical physics which still generates many questions in the theory of large random matrices.

Currently, random matrices and graphs are popular models for describing complex high-dimensional systems.

The course will present some standard tools: Gaussian processes, concentration inequalities, Stieltjes transform as well as emblematic results of the field: non-asymptotic study of extreme eigenvalues of symmetric Gaussian matrices, Wigner and Marchenko-Pastur theorem, large covariance matrices, perturbative models.

The following books provide excellent references but the course is self-sufficient. A handout partially covering the course will be distributed to students.

Keywords

Foundations of the theory of large random matrices - non-asymptotic study - global regime - local regime - correlated and uncentered models.

1. Linear algebra reminders, complex analysis, solver, Stieltjes transform.
2. Estimation of variances, Efron-Stein inequality, Poincaré-Nash inequality.
3. Gaussian processes, comparison and applications to random matrices.
4. Non-asymptotic study of the smallest and largest value of Gaussian matrices, Study of the non-Gaussian case using network methods.
5. Machenko-Pastur theorem
6. Large covariance matrices - canonical fixed point equation - application to models “spikes”.
7. Study of the non-centered “signal plus noise” model.

Bibliography:

— GW Anderson, A. Guionnet, and O. Zeitouni. An introduction to random matrices, volume 118 of Cambridge Studies in Advanced Mathematics. Cambridge University Press, Cambridge, 2010

— R. Vershynin. High-dimensional probability: An introduction with applications in data science, volume 47. Cambridge university press, 2018.

— Mr. J. Wainwright. High-dimensional statistics: A non-asymptotic viewpoint, volume 48. Cambridge University Press, 2019

A1 Analysis tools and partial differential equations

First Semester

Teacher: Marco Cannone.

The goal of this course is to complete students' knowledge of analysis (functional, harmonic, etc.) and to introduce them to some useful tools for partial differential equations and multifractal analysis. The following subjects will be covered: — Additional information on Banach spaces: duality, weak topology. . .

— Analysis of L p spaces : some properties, interpolation, applications. . .

— Reminders and additions to distributions: regularization and approximation techniques, tempered distributions, Fourier analysis.

— Sobolev spaces, Sobolev injection, Rellich compactness theorem. Application of Sobolev spaces to PDEs. Principle of variational methods. Application to the Dirichlet problem, maximum principle.

— Introduction to Littlewood Paley analysis: construction, algorithms, basic examples. Characterization of functional spaces.

— Applications. The Navier-Stokes equations: variational approach of J. Leray and resolution by fixed point of T. Kato.

Bibliography:

— RA Adams, Sobolev Spaces, Academic Press, 1975.

— H. Brezis, Functional analysis, Masson, 1983.

— M. Cannone Wavelets, paraproducts and Navier-Stokes, Diderot 1995.

— F. Hirsch, G. Lacombe, Elements of Functional Analysis, Masson, 1997.

— W. Rudin, Functional Analysis, Ediscience.

— K. Yosida, Functional Analysis, Springer-Verlag, sixth edition, 1995.

A2 Geometric measurement theory and multi-scale analysis tools

First semester

Teacher: Stéphane Seuret

In this course, we will first see all the fundamental notions of dimension used in analysis: box dimension, packing dimension, Hausdorff dimension. We will apply these notions to the study of the multi-scale properties of so-called "self-similar" sets and fractal sets.

Secondly, we will construct continuous and discrete wavelet bases. Wavelets are an analysis tool that has many applications, particularly in signal and image processing. We will study some of them. We will demonstrate wavelet characterizations of classical functional spaces (Lebesgue, Hölder, Besov).

Finally, we will link the first two parts of the course by studying the scaling properties of generic functions in functional spaces.

The course will end, of the students' choice, with one of the following subjects: 1. Multifractal analysis of

remarkable functions: Functions of Bolzano, Riemann, Polya,
Davenport series 2.

Introduction to ubiquity methods for calculating the dimensions of limsup sets 3. Multifractal analysis of Lévy

processes, application to the Burgers equation 4. Multivariate analysis

Bibliography:

— I. Daubechies. Ten readings on wavelets. Society for Industrial and Applied Mathematics 1992.
— K. Falconer. Fractal Geometry: Mathematical Foundations and Applications. John Wiley & Sounds, 1993.
— Y. Meyer. Wavelets and operators. Hermann 1990.
— S. Seuret. Multifractal analysis and wavelets. New Trends in Applied Harmonic Analysis, Springer, 2016.

A3 Deterministic and probabilistic approximation methods for applications in stochastic and financial modeling

First semester

Teacher: Robert Eymard

Synopsis. The course will focus on the approximation of hyperbolic, elliptic and parabolic equations resulting from the Chapman-Kolmogorov and Feynman-Kac formulas for different stochastic processes.

We will discuss the theory of deterministic approximation of these equations (Hilbertian notions, Galerkin methods, convergence of approximations for the topologies implied by the models).

At the same time, we will compare numerically on different scales the quality of these approximations with those given by Monte Carlo methods.

These comparisons will be the subject of project work by the students, which will constitute half of the note.

The other half will be based on an exam on the mathematical properties of the methods deterministic approximations.

Bibliography:

— R. Herbin, Numerical analysis of partial differential equations https://cel.archives-ouvertes.fr/cel-00637008

— T. Gallouët and R. Herbin, Partial differential equations, January 17, 2019. https://www.i2m.univ-amu.fr/perso/raphaele.herbin/PUBLI/M2edp.pdf — N. Wicker, Simulation course stochastic, http://math.univ-lille1.fr/~wicker/Cours/coursSimulationAleatoire.pdf

A4 Analysis and metric theory of numbers, applications to the study of the performance of communication models

Second semester

Teacher: Faustin Adiceam

Description: Metric number theory is a branch of number theory which studies, from a probabilistic point of view, the properties of approximation of real numbers by rational quantities. This theory today maintains very strong interactions with numerous fundamental fields (eg, ergodic theory, harmonic analysis, measurement theory) as well as applied ones (eg, information theory, cryptography, problems modeling).

The aim of the course will be to present some of the most important results in this theory in order to study their implications in signal theory. More precisely, theoretical tools relating, among others, to fractal geometry and Fourier analysis will be developed to understand the approximation properties of real numbers. These approximation properties will then be used to evaluate the performance of communication networks in signal theory using an innovative approach that has emerged in recent years.

The course will thus highlight, with supporting examples, some of the aspects of the interaction between number theory and signal theory. The potential for exploiting this interaction still remains immense.

Bibliography:

— Number theory meets wireless communications. Edited by Victor Beresnevich, Alister Burr, Bo-bak Nazer and Sanju Velani. Mathematical Engineering. Springer, Cham, 2020. (Especially chapter 1).

— Bugeaud, Yann. Approximation by algebraic numbers. Cambridge Tracts in Mathematics, 160. Cambridge University Press, Cambridge, 2004.

—Falconer, Kenneth. Fractal geometry. Mathematical foundations and applications. Third edition. John Wiley Sons, Ltd., Chichester, 2014.

— Mattila, Perthy. Fourier analysis and Hausdorff dimension. Cambridge University Press (2015)

—Jafar, Syed. Interference alignment a new look at signal dimensions in a communication network. Found. Trends Commun. Inf. Theory 7 (1) (2011).

A5 Introduction to Gamma-convergence

Second semester

Teacher: Vincent Millot

The object of this course is to present the main concepts of Gamma-convergence theory and to describe its applications in the asymptotic study of variational problems. In a first part, we will present the abstract theory of gamma-convergence and the main results on the convergence of variational problems. In a second part, we will illustrate the theory in the context of one-dimensional variational problems including as examples relaxation, homogenization, limits of discrete systems, or even phase transition problems.

Keywords: calculation of variations, gamma convergence, relaxation, homogenization.

Bibliography:

— A. Braides, ÿ-convergence for beginners. Oxford Lecture Series in Mathematics and its Applications cations, 22. Oxford University Press, Oxford, 2002.
— G. Dal Maso, Introduction to Gamma-convergence. Springer.
— E. Giusti, Direct Method in the Calculus of Variations. World Scientific.
—RT Rockafellar Convex Analysis, Princeton University Press.

A6 Partial differential equations and fractional Laplacian

Second semester

Teachers: Rejeb Hadiji

This course introduces the basic tools for the study of variational analysis of nonlinear problems described by nonlocal operators. In recent years, particular attention has been paid to these problems, both for their mathematical interest and the questions they raise, but also for their concrete applications to the modeling of numerous physical phenomena.

The fractional Laplacian is part of this family of nonlocal operators, it can be defined by Fourier analysis, functional calculus, singular integrals or Lévy processes.

In this course we will address: —

fractional Sobolev spaces — the fractional Laplacian

operator — fractional Sobolev spaces adapted

to the study of the operator (ÿÿ)s with a condition by Dirichlet.

— the analysis of fractional elliptic problems having subcritical nonlinearities, via classical variational methods as well as some results concerning critical fractional equations

— physical applications, allowing you to see how these various subjects are linked to other fields such as topology, functional analysis, mathematical physics.

This course is therefore aimed at Master 2 Mathematics and Applications students interested in EDPs and their many applications.

Bibliography:

— RA Adams and JF Fournier, Sobolev spaces, Second edition, (2003).

— M. Bonforte, Y. Sire, JL Vazquez Existence, uniqueness and asymptotic behavior for fractional porous medium equations on bounded domains, 35(12), 5725-5767, (2015).

— A. Hubert, R. Schäfer Magnetic Domains, The analysis of magnetic microstructures, no 18, Springer, (2009).

— G. Molica Bisci, V, Radulescu, R.Servadei Variatianal method for nonlocal fractional Problems, Cambridge University Press, (2016).

ACT Empirical estimation - extreme values

First semester

Teacher: Emmanuel Clément

Objectives (in terms of know-how):

— Model maxima and threshold crossings, — Estimate extreme quantiles.

Brief program:

— Order statistics, quantile estimation, quantile/quantile diagrams.
— Convergence of maxima (univariate framework), domains of attraction, GED laws.
— Threshold exceedances, GPD laws.
— Estimation of extreme law parameters, block maxima, threshold crossing, Hill and Pickands estimators, applications with R.
— Point processes and extreme values.
— Multivariate framework.

Skills acquired (direct/indirect):

— Asymptotic behavior of maxima and threshold crossings and applications to management risks
— Analysis of data sets with R software: estimation, tests, choice of models

ACT Anonymization and confidentiality

Second semester

Teacher: Claire Lacour

The objective of this course is to —
understand the problem of data confidentiality — know the role of randomness in confidentiality — reconsider the usual methods in statistics with the perspective of anonymity

The course program will be as follows:
— Introduction to the issue of data confidentiality
— Definition of differential privacy and main methods
— Differential confidentiality and statistics:
  — confidential estimate of the distribution
  — confidential parameter estimation — minimax study for local confidentiality

X Probabilistic and statistical modeling for epidemiology

Second semester

Teacher: Viet Chi Tran

The purpose of this course is to introduce and study probabilistic models of the propagation of epidemics in populations, as well as their deterministic counterparts. We will also address the issue of data and their statistical processing.

In the first part, we will present a compartmental model: the SIR model, a formulation of which Kermack and McKendrick studied using ordinary differential equations at the beginning of the 20th century. We will in particular introduce the probabilistic tools involved to describe and analyze it (counting process, limit theorems and approximation by branching processes, definition of the reproduction number R0 for this model, etc.) Then, we will address the statistical problems associated with it (estimation of parameters, forecasts, calculation of rare probabilities, etc.). A recurring problem in epidemiology is that the spread of the epidemic is often only partially observed.

In a second part, we will address refinements of this compartmental model: introduction of structure (for example, age or geography), household models. We will also study models where the epidemic spreads along a social network. A foray into graph theory and statistical questions associated with the reconstruction of these structures will be made.

Bibliography: —

F. Ball, T. Britton, E. Pardoux, C. Laredo, D. Sirl, VC Tran. Stochastic Epidemic Models with Inference. Springer. Mathematical Biosciences subseries,Vol. 2255 (2019).